Math 211 - Bonus Exercise 10 (please discuss on Forum)

1) Show that any group of order 30 has a normal subgroup isomorphic to $\mathbb{Z}/15\mathbb{Z}$.

2) Use the previous problem to classify (up to isomorphism) all groups of order 30.

3) Let $G = SL_2(\mathbb{F}_3)$, i.e. the group of 2×2 matrices with entries modulo 3 and determinant equal to 1. Show that $Z(G) \cong \mathbb{Z}/2\mathbb{Z}$ and that $G/Z(G) \cong A_4$. Hint: see last week's last bonus problem.

4) Find a Sylow p-subgroup of S_p for an odd prime number p. Then do the same for S_{2p}.

5) Find a geometric reason for why the icosahedral group is isomorphic to A_5 (forget for a second that we're in a class on abstract group theory and imagine that you're an ancient Greek mathematician contemplating the icosahedron).